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AUTHOR	Sassi, Annette M.; Nelson, Barbara Scott
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ABSTRACT

This paper illustrates how the instructors in a professional development program for school and district administrators used a range of discursive moves to help administrators cultivate a different appreciation of mathematics classrooms, and learn to pay attention to different features of a mathematics lesson. The way in which the seminar was taught was analyzed in an effort to understand what it was that helped the administrators make dramatic changes in their interpretation of the same videotaped classroom. The discourse of the seminar was examined with an eye to discerning the way in which instructors interacted with participants to help them learn to interpret the videotapes of reformed classrooms differently. After describing research methodology and theoretical orientation, four vignettes from the seminar were analyzed in which facilitative moves were made for the purpose of helping administrators attend differently to the events depicted on the classroom videotapes. (Contains 26 references.) (ASK)

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Learning To See Anew: How Facilitator Moves Can Reframe Attention When Administrators Look at Reformed Mathematics Classrooms

By
Annette M. Sassi
Barbara Scott Nelson

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Learning to see anew:
How facilitator moves can reframe attention
when administrators look at reformed mathematics classrooms[1]

Annette M. Sassi
Barbara Scott Nelson
Education Development Center

Paper presented at the annual meeting of the American Educational Research Association
Montreal, April 1999

Introduction

The current mathematics education reform movement, with its emphasis on mathematical thinking and reasoning in classrooms, poses new challenges for those who observe classrooms with the purpose of evaluating teaching. The emerging changes in mathematics classrooms are based on fundamentally different assumptions about what knowledge is and what teaching and learning entail (NCTM, 1989; 1991; 1998). Earlier, learning was taken to be the absorption of knowledge dispensed by teacher or text, and classroom observers looked for evidence of dispensing and absorption (for example, clear explanation of new ideas, opportunities for students to apply new ideas or practice new skills, students observably "on task" throughout the lesson). Now, however, mathematical thinking itself is the focus of classroom activity, and learning is taken to be the opportunity for students to do mathematical thinking in order to build their knowledge. In order to assess the quality of mathematics teaching (and learning) that is happening in such classrooms, supervisors and other classroom observers need to be able to discern the central intellectual ideas of the lesson and pay attention to how classroom structures and practices provide students with the opportunity to develop those mathematical ideas. If observers use observation instruments and habits of mind that are oriented toward teacher-centered classrooms (c.f. Hunter,

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1984), reformed classrooms are likely to look chaotic or out of control. If they use instruments and observational practices that are more student-centered but still emphasize low-inference, observable behaviors that are not subject-specific (c.f. Glickman, 1998), these observers are likely to recognize some familiar behaviors on the part of both teachers and students but miss the intellectual import of what is going on.

We have earlier reported on how a group of administrators dramatically changed the way they interpreted the elementary mathematics teaching depicted on videotape (Nelson & Sassi, 1998; Nelson, Sassi & Driscoll, 1999). These administrators viewed and interpreted the same videotape of a reformed mathematics classroom at the beginning and end of a year-long course in which they were learning about the fundamental tenets of mathematics education reform and familiarizing themselves with what reform classrooms might look like. At the beginning, they attended to aspects of the class characteristic of the process-product school of research on teaching, which emphasized low-inference, observable teacher and student behaviors that were associated with high student outcomes on standardized tests (wait time, highly structured lessons, good transitions between lesson segments, assuring that students understand before moving to the next portion of the lesson, etc.) (Brophy & Good, 1986). At the end of the course administrators were attending to aspects of the class much more in tune with the orientation of the mathematics education reform movement, in which the ideas in the subject, and the way those ideas are worked with, are what matters (Shulman, 1986).

In this paper we analyze the way the seminar was taught, in the effort to understand what it was that helped these administrators make such dramatic changes in their interpretation of the same videotaped classroom. The general pedagogical objectives of the seminar and its overall design have been described elsewhere (Nelson, Davidson & Sassi, 1998; Nelson, 1999). In short, the course was run according to the pedagogical principles that inform reformed mathematics classrooms. Administrators had direct experience in doing mathematics in order to understand the mathematical ideas, and viewing and discussing videotapes of reformed instruction. They used those experiences as the grist with which to think about new ideas about mathematics, teaching,

and learning. Here we will examine the discourse of the seminar, with an eye to discerning the way in which instructors (here called facilitators) interacted with participants to help them learn to interpret the videotapes of reformed classrooms differently. After describing our research methodology and theoretical orientation we will analyze four vignettes from the seminar in which facilitative moves were made for the purpose of helping administrators attend differently to the events depicted on videotapes of classrooms.

Methodology

Participants.

This was a self-selected group of administrators. Twenty-four administrators had been invited to participate in the seminar; 18 attended regularly over the course of the year. Participants included two assistant superintendents of curriculum and instruction, four district-level elementary mathematics coordinators, and twelve principals of elementary schools from four districts in a major metropolitan area, including several administrators from the city's school system. One participant was concurrently a high school mathematics teacher. The others had little formal mathematics training: the elementary principals would have taught mathematics as part of their elementary school teaching experience. Three had Ph.D. degrees, most had masters degrees in education.

There were four facilitators. Two were mathematics educators with substantial experience facilitating elementary mathematics professional development programs for teachers; one had recently been a high-school mathematics teacher; and one was a sociologist with particular interest in the nature of professional development.

Research Design.

This was a naturalistic inquiry in which we studied a year-long seminar for school and district administrators on classroom observation and teacher supervision in elementary mathematics. We had previously discovered that the ideas that administrators had about the nature of mathematics, teaching, and learning influenced the way they understood mathematics education reform and their

ideas about how to support it (Nelson, 1998), and that administrators could change the way they interpreted a videotape quite dramatically from beginning to end of the course (Nelson & Sassi, 1998; Nelson, Sassi & Driscoll, 1999). In this study our research question was: What did the facilitators of this course do to make it possible for participating administrators to change their interpretation of the videotape so dramatically?

For this work we were both teachers and researchers, in the genre of Lampert (1990), Ball (1993), and Hammer (1995). In this kind of work, the teacher/researcher collects data about the teaching process in any of several ways — by videotaping or audiotaping the classroom, retaining written student work, or keeping detailed teaching journals. While the data may be analyzed on a class-by-class basis to inform teaching decisions, for research purposes all data are analyzed after the class is finished for general patterns and relationships.

The seminar was designed to simulate actual classroom observation experiences and took place monthly through a school year (there were ten class sessions). At each session we showed a short videotape of a classroom in which the teacher was in the process of transforming his or her mathematics instruction in accord with reform tenets.[2] At each session administrators did and discussed the mathematics that would be presented in the lesson. They then viewed the videotape twice. After the first viewing they were asked, "What did you see," in answer to which they discussed what they perceived as salient about the mathematics and the pedagogy of the lesson. After the second viewing they were asked a focus question designed to draw out the kinds of things about which it would be important to talk with the teacher, and the pedagogy of the supervisor-teacher relationship.

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[2] Many of these videotapes were produced by the Educational Technologies Department of Bolt Beranek and Newman Inc. in a project entitled "Mathematical Inquiry through Video: Tools for Professional Growth," supported by the National Science Foundation. The videotape used at the beginning and end of the seminar depicted Magdelene Lampert teaching a fifth grade class in November, 1989. It was made as part of a project entitled "Communication In and About School Mathematics," funded by the National Science Foundation (grant # TPE8954724).

As facilitators, we intended these discussions to encourage administrators to articulate and examine their own understandings of learning, teaching, and mathematics — ideas that many administrators had had for so many years that they functioned as assumptions and were no longer critically examined. All four of us shared the task of facilitation. At any given moment, one of us was primarily responsible for the instruction, according to pre-arranged plan. However, since we had planned these sessions together, all four of us were aware of the purposes of the instruction at all times, and all four participated in the conversation, from a facilitative stance, throughout each class session.

All seminar sessions were audiotaped, and the tapes were transcribed. Ethnographic field notes were taken at all class sessions. This paper is based on analysis of the transcripts and ethnographic field notes of all ten class sessions, with particular attention to the facilitators' comments. Each researcher independently read and coded the data. We first analyzed the data to develop a list of facilitative moves that were made. We then analyzed the data about each of the major ideas that changed (nature of mathematics, learning mathematics, student engagement, and nature of teaching), noting the way previously identified facilitative moves functioned in each area. For both analyses, we check-coded to identify inter-observer reliability (Miles & Huberman, 1994). In analyzing the transcripts, categories emerged from the data, as in grounded theory research (Glaser & Strauss, 1967). Preliminary categories were identified and the data was re-read to refine or disconfirm those categories.

Constructing New Ideas: Using Facilitator Moves to Reframe Attention

When done well, facilitating a discussion appears seamless and the skill required is transparent. But the task of the facilitator is more than just ensuring that the discussion moves along and appropriate topics are addressed. In this seminar, the challenging task facing us was to shift the administrators' focus in the videotaped classroom from attention to surface behaviors to attention to the mathematical ideas at play.

Our research on administrators' observations of teaching is theoretically situated in the study of practical judgment, which is concerned with the reasoning and judgment involved in taking practical actions (Fenstermacher & Richardson, 1993; Pendlebury, 1995). Experienced administrators exercise such judgment regularly. For instance, in observing in classrooms administrators make subtle decisions about what it is important to attend to – which facts matter – for the purpose of making a practical judgment about the adequacy of the instruction. Administrators interpret the significance of such things as the noise level of the classroom, which students are called on, the amount and kind of student activity, or any of a wide range of teacher behaviors. Decisions about the facts that matter also help shape the related practical activities in which administrators engage — the notes they make about what they observe, what they say when they consult with the teacher before or after the class, what recommendation for further action they make. We have adopted Vickers' (1965) term "appreciation" for these acts of simultaneous perception and interpretation in which fact and value are intermingled.[3] Our work with administrators in the seminar essentially entailed helping them shift from viewing and interpreting the videotapes of classrooms through one appreciative system to viewing and interpreting them through another. We were helping them change their sense of the facts that mattered.

During the period when most current administrators were educated, the process-product program of research on teaching was dominant, affecting the design of textbooks and training programs in teacher supervision and the design of instruments for use in classroom observation (c.f. Hunter, 1984; Darling-Hammond & Sclan, 1992; Reitzug, 1997). Dimensions of teachers' behavior that were thought important included the pacing of instruction; the presence of wait time after questions are asked; the structuring of lessons; frequency of praise or criticism; use of lower-

[3] In studying the nature of judgments entailed in corporate and governmental policy-making Vickers focused on what he called "appreciations" — ways of noticing what is going on that relate the facts to what is thought to be important. Vickers argued that in assessing a situation in order to make a practical decision, judgments of fact and value are inextricably interwoven. Further, in his view, at any given moment in time, a person's appreciative system has a setting — "a set of readinesses to distinguish some aspects of the situation rather than others and to classify and value these in this way rather than that" (Vickers, 1965, p. 68).

or higher-order questions; and so on (Brophy & Good, 1986). While current textbooks in supervision are beginning to address observation of classrooms that are more student-centered than in earlier years, the observation strategies and skills still focus principally on observable behaviors on the part of teachers and students. (See for example Glickman et al., 1998). In this view of classroom observation and teacher supervision, the content of the observation is overtly observable teacher and student behaviors, and the strategy for observation is to note the presence or absence of certain categorical items that had been shown to have statistical significance. An administrator oriented in this way has a readiness to appreciate – to notice because he values – these aspects of the classrooms being observed.

However, current mathematics education reform efforts are built on the notion that the ideas in a subject, and the ways in which students and teachers work with the ideas, matter (Shulman, 1986). That is, classroom observation requires that supervisors pay attention to the mathematical content of the lesson, the nature of the students' ideas about that content, and the opportunities provided for students to deepen their understanding of those ideas (Nolan and Francis, 1992). Rather than simply looking for a more contemporary set of categorical elements of instruction – manipulatives, small group work, mathematical discussion – as indicators of good instruction, administrators will need to pay attention to how these elements are used to support the development of mathematical thinking.[4] Now, the content of the observation is mathematical ideas and the strategy for observation is to understand how the teacher's decisions support the development of mathematical ideas on the part of the students. An administrator oriented in this way has a readiness to appreciate – to notice because he values – different aspects of the classrooms being observed.

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[4] Darling-Hammond (1992) makes the point that in behavioral approaches such as "process-product" and "effective teaching" research, some behaviors found effective in some situations are ineffective or counterproductive in others. This point emphasizes the need for supervisors to not focus solely on the behaviors but to understand the situation and the context in which the behaviors are being employed in order to discern if they are appropriate for the situation.

Central to making the shift of attention we were aiming for was a shift in administrators' understanding of what learning, teaching, and mathematics entailed. Such conceptual shifts are difficult (c.f. Carpenter, et al., 1998; Schifter & Fosnot, 1993; Schifter & Simon, 1992) and many aspects of the seminar addressed this goal (see Nelson, Davidson, & Sassi, 1998; Nelson, 1999). As we reported earlier (Nelson & Sassi, 1998), as administrators in the seminar came to understand mathematics, teaching, and learning differently, they could see different things in the videotapes. For example, one administrator thought, on first viewing a videotape of an elementary mathematics lesson in October, that the teacher was likely in the process of transition from a didactic style of teaching to a style more oriented toward teaching conceptual ideas in mathematics. He said,

> I see her as somebody in transition, going from a teacher-centered, ... didactic style of teaching to somebody who wants to be working on more higher order thinking skills. ... Her thinking was, I thought, very scattered. ... You're seeing all the evidence of somebody who's having a hard time focusing.
>
> October 3

On the second viewing of the same tape, in June, he saw her as a very good teacher; one who could engage in both direct teaching and the facilitation of students' thinking and, furthermore, could purposefully choose which style to use. He observed:

> I was seeing both things [direct teaching and exploring]. She has intentional moves that were very direct when she needed to [use them], so I think that there's a strong teacher there.
>
> June 12

By June, this administrator had become more experienced at watching reformed classrooms and had had many opportunities to consider the idea of mathematical exploration and what it looked like in an elementary mathematics classroom. What he had earlier defined as unfocused and scattered behavior on the part of the teacher he now interpreted as the purposeful facilitation of mathematical exploration. He saw because he understood.

Moreover, the processes of developing new ideas about mathematics, learning, and teaching, and seeing videotapes differently seem to be iterative. The very concreteness of the

images on the videotape made it possible for members of the group (and the facilitators) to point to incidents on the tape and interpret, or reinterpret, them from the viewpoint of different appreciative systems. Reciprocally, seeing an incident on the videotape interpreted as an example of, say, mathematical exploration, gives a concrete image for that concept. As we analyzed the data about the facilitation of this seminar, we came to see that facilitative moves were importantly a matter of interpreting and reinterpreting incidents on the videotapes.

Facilitating a professional development seminar, like teaching in a classroom, requires a repertoire of strategies combined with a range of competencies and abilities that result in being able to make facilitator moves. A move is a complex action involving some type of deliberation, evaluation, choice, and some type of practical or visible activity that may involve doing or saying something or not doing or saying something. A move may entail asking a question, choosing to listen, or opting not to intercede when students may be struggling.[5] In the case of facilitating a whole group discussion in a professional development seminar like we ran, facilitator moves were often visible as discursive moves — specific discursive actions that the facilitator made.

Some of the more common discursive moves used in the seminar that contributed to reinterpreting an aspect of the videotape according to a different appreciative system include:

> 1) Paraphrasing, repeating, or summarizing an idea (sometimes in the process of writing things on flip chart paper). This had the effect of calling attention to particular ideas and giving them more "airtime." It also occasionally provided the context for subtle re-wording which had the effect of raising a new meaning for an idea;
>
> 2) Agreeing with a participant's interpretation. Usually, when a participant offered an interpretation that was consonant with a reform appreciation of the class, this facilitative move would be made.
>
> 3) Asking for specificity or elaboration of an idea. This prompted the explication of more, and often richer, detail about administrators' ideas and legitimated the notion that their ideas were germane. When done in reference to an incident on a videotape, this often focused attention on a different level of detail on the videotape than the speaker had originally intended and legitimized paying attention at that level;

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[5] See Ball, 1997, for an evocative description of elementary mathematics teaching that describes in similar terms the decisions and moves that teachers make.

4. Challenging an idea. By asking a question, pointing out a counterexample, or offering an alternative explanation of an incident on the videotape, this move challenged a participant's interpretation of the videotape and offered an alternative for consideration by the group;

5. Offering one's own observation. This had the effect of putting a new idea on the table for discussion;

6. Reframing. This move reinterpreted some aspect of the videotape in terms of the reformed appreciation of mathematics instruction. The switch of perspectives often involved highlighting something that had been ignored before, or pointing out something as valuable that had not been attended to before.

At the core of such facilitation is understanding the critical ideas that drive the discussion and knowing how and when to use which moves to forward these ideas. We have suggested that in order to shift their appreciative systems, administrators needed to grapple with ideas in four substantive areas: the nature of mathematical knowledge; how mathematics is learned; the nature of student engagement; and the nature of teaching (Nelson & Sassi, 1998). Facilitative moves in this seminar were employed strategically to help administrators grapple with these ideas, as there were opportunities to do so. The next section of this paper will look at four vignettes, drawn from across the sessions, one from each substantive area, to illustrate how the different discursive moves were embedded in the context of shifting administrators' attention in classrooms and helping them to cultivate new understandings of mathematics, learning, and teaching.

Learning to Appreciate Mathematics Classrooms Anew

The four vignettes discussed below are drawn from four different seminar sessions across the year. Each is drawn from a different strand of administrator learning — coming to see mathematics as being about ideas, how mathematics is learned, the nature of student engagement, and the nature of teaching — and each illustrates how a facilitator used different discursive moves to help administrators attend to the videotape with a new eye.[6] In each vignette one can see the

[6] It would be interesting to look at how the facilitation of these video discussions fits more broadly into the use of cases as teaching tools. Central to the use of cases, whether written or video, is the ways in which they can help people to cultivate situated judgment and perception.

iterative movement between new ideas about the facts that matter and attention to different aspects of the class depicted on the videotape.

The first vignette, an illustration of cultivating an understanding of mathematics as about ideas and ways of thinking, looks at the way in which the facilitator highlighted a sixth-grade teacher's use of a counterexample to encourage administrators to move beyond listening only to the teacher's questioning techniques and attend also to the teacher's mathematical reasoning. The second vignette illustrates a facilitator working to shift administrators' understanding of the ways mathematics is learned — from accumulating discrete chunks of knowledge that can be learned from clear explanations to developing conceptual understanding over time — by offering a different interpretation of a confusion evident in a sixth-grade class exploring angles. The third vignette, an illustration of shifting administrators' ideas of what counts as student engagement, looks at how a facilitator helped the group reconsider their initial interpretation that a fifth-grade teacher's interruptions signaled gender-biased behavior. The fourth vignette considers how two facilitators together helped the administrators to reconsider the nature of teaching by examining the possibility that a sixth-grade teacher's "good timing" could also be interpreted as the fostering of a culture of inquiry in the classroom.

In each vignette, the names of the administrators have been changed but their job titles and gender have been retained. Facilitators are indicated by italics. In three of the four vignettes, a facilitator other than the one leading the discussion also spoke. In these vignettes, the lead facilitator is indicated as "facilitator #1" and the other is indicated as "facilitator #2." These are different people in each vignette.

What counts as mathematics knowledge.

With the recent push to reform mathematics education, what counts as mathematics knowledge has begun to be redefined. While knowledge of facts, procedures, and correct answers still do matter, mathematics education has shifted away from the rote memorization of facts and procedures to

emphasizing the interconnectedness of mathematical ideas and constructs, and introducing students earlier to the inherent ambiguities and complexities of mathematical systems.

What this means for school administrators is that in many cases their own understanding of mathematics has to shift in order to make sense of the pedagogical shifts that are taking place in classrooms. Administrators can relatively easily identify the pedagogical shifts, as we will see in the excerpt below. But being able to understand and fully appreciate the new pedagogies requires administrators to understand mathematics not as a set of discrete operations and mathematical facts but as mathematical ideas and concepts that need to be reasoned out and made sense of.

Reshaping administrators' attention in mathematics classrooms to pay attention to the mathematical ideas at play involves, among other things, helping them attend to and think about the mathematical reasoning of both the teacher and the students. In the more traditional paradigm, making sure that students understand how to do procedures and are able to execute them is what counts as "knowing mathematics." In more contemporary paradigms, being mathematically knowledgeable entails being able to reason mathematically, understand how a procedure works, and make sense of the mathematical concepts that underlie the procedures and algorithms. These shifts in understanding underlie and interact with administrators' ability to appreciate what is happening on a videotape in new ways.

The following excerpt is taken from a late October session in which the administrators observed a sixth-grade class work on the problem, "What is 5% of 40?" In the class, four of six student groups got the answer 8 while the other 2 groups got the answer 2, which is the correct answer. The students who got 8 did so by incorrectly dividing 5 into 40. At one point, the teacher tried to lead the students into exploring why the strategy was wrong by using an extreme counter-example: Asking if 40% of 40 would be 1. [7]

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[7] The students' error was to divide 5 into 40, getting the answer 8, rather than understanding that they should convert the percentage to a decimal and multiply .05 x 40, to get the answer 2. Alternatively, they might have understood that 5 percent is one-twentieth of 40, and they should divide 20 into 40, or multiply 1/20 times 40, to get the answer 2. In the counterexample, the teacher did the same mathematical operation as the students who had made the error, but with numbers that he thought

In viewing the video, the administrators were given a hypothetical scenario: they were told that the teacher was working on understanding the students' mathematical thinking and that they should look to see if there is evidence that he is doing that. The excerpt picks up with Mark, an elementary school principal, describing the teacher's questioning techniques:

> Mark: So, I think one lens that you can [take] on [his understanding of students' mathematical understanding] when you're discussing with him is [his] questioning techniques: "You've said [attending to students' mathematical thinking] is important, so let's look at the evidence." And, it starts right away: "I want to see what we've got." So, [the question] puts it back out there. And then [you] go through the litany of questions that he asks, that literally say [to students], "What are you thinking?" [If I] want to know what you're thinking, ask. He's asking.
>
> *Facilitator #1: You're saying that if you look at how [the teacher] behaves, you get a sense of what he's paying attention to?*
>
> MM: Yes. And then, so you can cite all of those [questions that he asks]. And then you could go back and cite .. I starred some of them, just circling [on the transcript] lots of the questions where he's asking – "Could you show us?" I don't understand what you said." "I heard the word 'minus' and I'm not quite sure where that comes from." So again, asking kids to explain their thinking. And then there are some sections where he's playful in the question – "I'm still wondering about Elise," and the kids laugh. I didn't get a sense that they're laughing at her, they're laughing at him being stuck. So, there's a playful nature: "I'm stuck on that." So, he's modeling "stuck." He's modeling his own thinking which suggests that they should be, and then he models thinking out loud: "That looks like just what Elise had, only a little different." So, by modeling thinking out loud he's encouraging that. And then he rephrases: "So, you said you were thinking of some number that goes into 40 five times." ... Then he gives .. he illustrates his confusion over "let me trying something here" and he goes to the board: "I'm hearing people say that they have forty left and they're supposed to take 5% of it." So by modeling it, and kind of blowing out the confusion, he doesn't leave them stuck there. He challenges that. Then there's a place were I made a note, like he's closing in on the truth."
>
> *Facilitator #1: I was interested in that 40%. I was trying to figure out what exactly was he doing there? The first time I saw it, I got really confused – when he says, what if you were taking 40% of 40? Mathematically, what was he doing?*
>
> Carol: He's trying to show them the flaw in their thinking.
>
> *Facilitator #1: But, how did that show them the flaw in their thinking?*
>
> Donna: I don't think he was showing – because the business confuses some people.
>
> *Facilitator #2: I think he just thought of it off the top of his head.*
>
> Donna: I think it was the reasonableness of it.

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would make it obvious that the method is wrong. If one calculates 40% of 40 by dividing 40 into 40, one would get the answer 1. But by using one's number sense one can see that 40% of 40 (nearly 1/2 of 40) would have to be bigger than one. Therefore, the procedure must be wrong.

Judy: What the children showed when they got 8, he was taking 5% by dividing the 5 into 40. And I think he thought that the children would know that 40% of 40 should be 16 and not 1. So, if he did the division and showed that that process wasn't working, wasn't correct. But, I felt that there was a little something left hanging there that wasn't clarified to the kids even if his examples .. I'm not sure that every student understood what he was doing when he illustrated those particular numbers. He did 40 and 40. Did he do another?

Eve: 50

Judy: 50

Facilitator #1: So, what he was doing, he was doing the thing that they did with a set of numbers where common sense would lead you to .. is that what he was doing?

Judy: That's what I thought he was doing. But, I wasn't sure that every student knew what he was doing.

The focal facilitation move in this excerpt, which begins with "I was interested in that 40%," has the primary facilitator *offering one of her own observations*.[8] More specifically, in strategically offering her observation, the facilitator highlights a different salient issue than had been mentioned so far and revalues its significance. We can look more closely at what happens here and how this move helps to focus administrators' attention on the importance of the mathematical ideas and the mathematical reasoning behind the teachers' pedagogical moves.

The excerpt begins with Mark, an elementary school principal, describing his observations of the teacher in the video. Mark had been paying attention to the kinds of questioning techniques the teacher used and had made some careful notes in documenting these. He used the facilitator's question about students' understanding as an opportunity to share some of the teacher's questions: "Could you show us?," "I heard the word minus and I'm not quite sure where that comes from," "So, you said you were thinking of some number that goes into 40 five times."

For Mark, these questions were the "evidence" that the teacher was attending to the students' thinking. While they are the types of questions that are encouraged in new pedagogical approaches, Mark stayed focused on the way in which these questions "modeled" a particular type of mathematical discourse and did not consider the actual student thinking that they elicited.

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[8] The names of facilitative moves are italicized, referring to the list of facilitative moves presented earlier in this paper.

As a way of bringing the focus of the discussion back to mathematical content, the facilitator picked up on Mark's alluding to the teacher's use of a counter example: "Then he ... illustrates his confusion over "let me try something here," and he goes to the board." Out of all of the points that Mark made, she decided to focus on this one by noting her own observation about it. She said, "I was interested in that 40%," and then she asked, "Mathematically, what was he doing?" The initial responses suggest that it was not obvious to the administrators what was mathematical about the counterexample. One administrator noted that it was an attempt to show the "flaw in their thinking." The facilitator then tried to pushed on the point a little more by asking for more detail in how this could expose a flaw in student's thinking. This move eventually led a mathematics supervisor, Judy, to talk about how the use of the counterexample would show the students that their procedure wasn't working because the answer to 40% of 40 could not be 1.

While it is evident that the administrators still were not fully facile with discussing the mathematics, the facilitator's move to highlight the counterexample aimed to direct administrators' attention to the nature of mathematical reasoning and the cultivation of number sense evident in the classroom on the videotape. The teacher in the video was trying to engage the students in mathematical reasoning and calling upon their own number sense to uncover what was not correct in their initial approach to the problem. He was using a common element of mathematical discourse — pointing out a counterexample — to do this. But the first administrator hadn't appreciated the teacher's actions in this way and the other administrators' struggled to see this in the videotape. The vignette ends with Judy, who understood what a counterexample was and could see it in the discourse on the tape.

Both mathematical reasoning and number sense are critical components of a reform conception of mathematical knowledge and it is necessary for administrators to appreciate (see because they value) them when they occur in classrooms they are observing, not notice only the nature of a teacher's questioning techniques without regard for the mathematical thinking that those questions reveal. This vignette illustrates the facilitator's effort to help administrators see these

concepts enacted on the tape by offering an observation that called attention to something they had not noticed and working to reinterpret the action on the tape in terms of a new appreciative system.

How mathematics is learned

To be able to fully appreciate the redefinition of what counts as elementary mathematics knowledge, administrators had to grasp that the ways one learns mathematics has to shift. Traditionally, educators assumed that mathematics could be taught as discrete pieces – mastering the algorithms for the operations; memorizing math facts; learning mathematical definitions such as what an angle or a line is. But, when mathematics is conceptualized as interconnected ideas and concepts, the learning of it happens more iteratively than linearly. Students' understanding develops over time and through different explorations. The learning of procedures, factual knowledge, and mathematical conventions is embedded in the mathematical investigations themselves.

One potentially confusing aspect of this learning approach is indeed the place of confusion. In classrooms where students are grappling with ideas and concepts, they are likely to feel confused from time to time, and making sense of their confusion can be difficult for both teachers and classroom observers. A typical tendency for classroom observers is to offer solutions to clear up what are perceived as misconceptions or confusions: "If only the teacher had said . . ." However, when teachers and administrators understand that confusion can be an indication that mathematical reconceptualization is going on, a more productive strategy can be to inquire into what is causing the confusion before jumping in to fix it.

The second excerpt is taken from an early December session in which the administrators observed a sixth-grade class using protractors to measure angles. The class had a somewhat chaotic feel to it, with students seemingly confused by both how to use a protractor and by such basics of geometry as what constituted an angle. Making sense of what was at issue for both the teacher and the students was challenging and posed the real temptation to offer quick pedagogical solutions. The excerpt begins when Carol, an assistant superintendent, questioned whether or not

the students' confusion could have been cleared up if they had been more directly instructed in the use of protractors. Her comment was followed quickly by a reflection by one of the facilitators about what could be confusing for students. This facilitation move helped to draw attention to the actual confusion itself, making it an object of inquiry.

> Carol: Wouldn't it have made some sense for the kids to have learned early on that .. you really have to .. when measuring an angle, it's critical to put this little circle ... and to measure from there. They weren't sure about that. It seemed to me that a lot of the confusion might have been cleared up if they knew the technical ..
>
> *Facilitator #2: I thought ... that they weren't clear about the ... the mechanics of using a protractor. ...And so then I started wondering if they weren't clear on the mechanics of using it, if they were willing to put all these ways, then what did that say about what it was they were thinking about size of angle? You know, the fact they were willing to do what looked random .. you know, what did that say about what they thought about the tool and what they thought about what measurement meant?*
>
> *Facilitator #1: Yea, so you're seeing a relationship between the mechanics of the measuring instrument and what they understand.*
>
> Janice: Going back to [the student in the video] and her sort of trying to figure out which way .. I'm a little bit torn because on one hand, you can simply say you always begin reading at zero. That's a pretty easy way to clear up that confusion. Not that they always do it but that .. that should do it. On the other hand, she was really .. she was thinking about it. She was using some judgment. She didn't have a rule but she was using judgment in terms of what does this look like. It looks like it's greater than a right angle so .. so this tells me something else. And, so it was sort of interesting.

The facilitation move in this segment entails a second facilitator *challenging* Carol's assessment of the situation.[9] She essentially intercepted Carol's thought that a "a lot of confusion might have been cleared up" if students knew the technical aspects of using a protractor by putting out her own question about what was at issue for students. She noticed that students were willing to use the protractor in all sorts of apparently random ways. This observation made her question what they even knew about angles in the first place. She might have been thinking, for instance,

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[9] Facilitator #2 in this segment was not the one primarily responsible for facilitating this particular discussion. In analyzing the data, we discovered that a non-facilitator would often play an implicit facilitation role, which we dubbed "ringer." That person would often express thoughts, ideas, or insights, much as a participant might but these could be viewed as deliberate facilitative moves. This observation raises some interesting research questions about the ways in which co-facilitating and co-teaching works.

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that students who had a clearer understanding of the nature of angles might have been more able to engage in a more mathematically-informed exploration of what a protractor was and how it could be used to measure angles.

The primary facilitator *paraphrased and summarized* the second facilitator's comment. In doing so, she opened up the space for Janice, an elementary-level assistant principal, to ponder the tension between clearing up students' confusion by telling them what to do and allowing them to explore as a way of using – and developing – their own judgment.

This brief interaction illustrates the way in which a facilitator could *reframe* a classroom event as a strategy for directing attention toward an alternate way of thinking about how students actually learn. It was a moment that offered the opportunity to continue a year-long exploration of the different meanings of confusion in traditional and reform conceptions of learning. Carol may have felt that the students in the video would have been able to measure angles more correctly if the teacher had instructed them in the proper use of the protractor. While this may have been the case, the secondary facilitator was willing to suspend this direct solution to consider a more mathematical issue – that students did not understand the properties of angles fully enough for such instruction to be as useful in clearing up the underlying confusions.

The nature of student engagement.

One classroom feature that an observer is apt to notice is how the students are engaged in the lesson. For administrators first learning how to observe in reform-inspired classrooms, their initial sense is often that students should all be actively engaged in the activity or discussion. Sometimes this translates into concerns for classroom fairness and equity — all students should be called on equally. No student should be called on too many or too few times. Teachers should balance calling on girls and boys. Students should not be left out of the discussion.

While the attention to equity issues in classroom involvement and student engagement has certainly sensitized both teachers and administrators to the unwitting ways in which some students can be systematically left out of the discussion, much can be missed if attention is paid only to the

quantifiable aspects of student engagement. The following excerpt illustrates how a facilitator helped administrators to look more deeply at the complex nature of student engagement and to reconsider their own initial interpretations of a teacher-student interaction as gender-biased.

The excerpt is taken from the first session of the seminar, held in early October.[10] The video segment shown in the session was of a whole group class discussion that primarily dealt with the problem, "If a car is traveling at 50 miles per hour, how far will it go in 10 minutes?" Several students had gotten the answer 5 by dividing 10 into 50. The administrators' discussion below is about an interaction between a student, Charlotte, who had gotten the answer of 7, and the teacher. As Charlotte described what she did to solve the problem, the teacher interrupted her several times: 1) to ask a student leaving the room to check whether or not other students were already out of the room; 2) to ask students to pay attention to Charlotte's explanation; and 3) to ask if someone else could explain Charlotte's strategy of making two parallel horizontal lines, one marked sixty minutes and one marked 50 miles. With the last interruption, the classroom discussion became collaborative and did not return to a one-on-one discussion between the teacher and Charlotte.[11]

In the excerpt, the elementary school principal, Mark, noticed the teacher's interruptions and interpreted them negatively. The facilitator then *offered her own observation*, that Charlotte was willing to say she had gotten 7. She then asked the group to think about why Charlotte was willing to raise her hand.

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[10] The video shown in this session was the same one discussed in Nelson & Sassi (1998) and depicts Magelene Lampert teaching a fifth grade class in November, 1989. It was made as part of a project entitled, "Communication in and About school Mathematics," funded by the National Science Foundation (grant #TPE8954724).

[11] While Charlotte's answer is incorrect (the answer is 8 1/3 miles in 10 minutes), the strategy of dividing the two lines into equivalent parts can help students recognize the complexity of the problem as well as derive the correct answer. By stopping the class to put Charlotte's method on the board and asking the other students to pay attention because they might later have to persuade Charlotte of *their* method, the teacher kept Charlotte's mathematical idea at the center of the whole-group discussion. Instead of viewing the interruptions as disrespectful to Charlotte, it is possible to view them as profoundly respectful, i.e. as an effort to keep Charlotte's mathematical idea at the center of attention, despite the need to attend to other matters. By so doing, the teacher could be viewed as communicating to Charlotte the value and importance of her mathematical thinking.

Mark: ... Then she also interrupted the little girl three times. She [earlier] made a decision to stick with the boy who was explaining his logic and [then] she interrupted the girl three times and never let her finish her logic. And, I think if I were Charlotte, I probably would have the message that what I think doesn't matter here and next time I'll just do something else with .. I won't my .. share my story.

Facilitator:: So, what made .. in your thinking about data. One of the pieces of data I saw here was that Charlotte was willing to raise her hand and say she had 7 when lots and lots of kids had said they had 5. So, what made Charlotte in this class feel like she could raise her hand and say .. I mean, she's been in this class for 2 months ... but she was willing to say that she had a different answer from every body else.. that had been this//

Mary: I think she's competent in math. She, herself, is competent in math. But, what the teacher was telling her, I wrote down, the teacher is telling her only boys are competent in math . even though she kept explaining, she got cut off 3 times. And she kept going back to the boy. And she said, and now we'll go on and see what so-and-so had to say [fades out here].

Donna: She did something that really bothered me .."this communicates to you" as opposed to us. But you did a really good job communicating this to us when she already told the kids they better listen cause they may have to paraphrase what she did or convince her otherwise .. which I thought was ..

Bob: Trying to conjecture. I got the funny feeling that she .. the kids felt comfortable to take risks//

Facilitator: Yeah//

Bob: What are our conjectures? Obviously that's very common. They made a lot of conjectures in there .. a lot of guesses.

Facilitator: So what made you feel like you saw evidence that this was a real [Bob still talking. Facilitator's last words inaudible.]

Bob: Even though I have some issues .. I think that it's encouraged to come with conjectures and to hypothesize... I think that's encouraged by the teacher. I don't know how well she's doing but I think she encourages it. Or else that girl would not have taken ...//

Facilitator: Well, yes, right. Because, I mean it seems that was fairly high risk .. for a kid to do in the classroom.

The main facilitation move in this segment was the facilitator's *offering of her own observation* in part as counterevidence to Mark's initial assessment of the teacher's interaction with Charlotte: "One of the pieces of data I saw here was that Charlotte was willing to raise her hand ..." In this case, she did not pursue the gender issue and, in fact, *changed the focus* to why Charlotte would raise her hand in the first place. In this move, the facilitator's counter-evidence to the gender-biased interpretation invited administrators to look beyond the immediacy of the interruptions at the classroom culture and norms.

The administrators slowly began to reconsider their interpretation. Mary, another elementary school principal, commented that she thought Charlotte was competent in math but still insisted that the teacher's reactions would have left her feeling not competent. Donna, a mathematics specialist, followed with a criticism of the teacher's tone of voice.[12] When Bob, an older, more traditional elementary school principal, made the tentative suggestion that the students were willing to take risks ("I got the funny feeling that ..."), the facilitator *picked up on his point and encouraged the line of thinking* with a simple, open-end, "yeah ..." When Bob noted that the class in general made a lot of conjectures, the facilitator *encouraged him to say more by asking for specific evidence*: "So, what made you feel like you saw evidence." He attempted to put into words why he thought that, ending with the observation that Charlotte would not have taken a risk if it hadn't been encouraged by the teacher. The facilitator immediately *reinforced this notion* by stating : "Well, yes, right. Because, I mean it seems that was fairly high risk .. for a kid to do in the classroom."

This brief vignette illustrates how the facilitator was able to reshape an initial interpretation of the situation by highlighting a different piece of classroom data. In doing so, she provided the administrators with some different ways to look for student engagement. Rather than looking strictly for quantifiable evidence such as how many times the teacher called on boys and girls or how many times she interrupted a student, the attention was put more on students' willingness to take mathematical risks and to share their ideas.

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[12] The teacher's second interruption was to get students to pay attention to Charlotte. She said, "I want to ask everybody to pay real close attention to this – because lots of people think the answer is five and Charlotte is explaining why she thinks the answer is seven miles. If you disagree with what she did, you're going to have to convince her that the answer is 5, so you better pay attention to this." Donna took issue with the phrase, "you better pay attention," arguing that that sounded too threatening.

The nature of teaching.

Teaching within reform pedagogies is viewed not solely as a means to transmit information but as the facilitation of students' learning and inquiry. It can be challenging for school administrators who observe in classrooms to understand what such teaching really entails and to perceive what such teaching actually looks and sounds like. As the first vignette illustrated, administrators might be apt to look primarily for specific teacher behaviors, such as questioning strategies or wait time, but not fully appreciate what the behaviors are allowing students to do. Shifting one's eye to consider the nature of teaching within the broader context of student learning requires one to pay attention to how the teaching moves allow students to engage in intellectual inquiry.

The last excerpt, from a session in late February, illustrates how two facilitators kept attention focused on how a particular teaching move contributed to the cultivation of a culture of inquiry within a fifth-grade class. It is taken from a discussion of a video in which students shared solutions to the following word problem:

> Amanda works on her science fair project for 2 1/3 hours on Friday and 1 3/4 hour on Saturday. Sally works 1 5/8 hours on Friday and 2 1/2 hours on Saturday. Who worked longer? How much longer did that person work? Use a picture to show how you got your answer.[13]

The video segment that administrators viewed depicted two students who had solved the problem by converting all of the fractions to their equivalent in 24ths before doing any of the addition. The students were at the board to explain their strategy to the rest of the class.[14] At one point, the explanation became quite confusing and the class become quite noisy as a number of other students started talking at the same time. One student commented that he didn't understand

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[13] The answer to this problem is that Sally worked 1/24 of an hour (or 2.5 minutes) longer than Amanda.

[14] The drawings that the two students put on the board were somewhat difficult for the rest of the students to understand. They depicted each of the whole hours by a rectangle shaded in with vertical lines. For each of the fractions in the problem, they divided a rectangle into 24ths according to a different set of multiples: 1/3 was shown on a 2 x12 array; 3/4 and 1/2 were each shown on 4 x 6 arrays; and 5/8 was shown on a 3 x 8 array. They had a total of 10 different rectangles on the board.

what the two students were doing. At that point, the teacher stopped the discussion and instructed the one student to reframe his comment to the two students as a question to be investigated. At one point, the explanation became quite confusing and the class become quite noisy as a number of other students started talking at the same time. The teacher had to stop the discussion entirely and rephrase the problem.

In the seminar excerpt, the administrators' discussion revolved around the particular point at which the teacher stopped the student from commenting in a way that might come across as criticism. Judy, the mathematics coordinator and high school math teacher, commented that she liked the teacher's timing of how long to let the discussion go on. The facilitator chose to focus the discussion not on the timing issue per se but on the teacher's move to have the student reframe the comment to the question, drawing attention to how a culture of inquiry is built.

Facilitator #1: Anything else? What else did others see?

Judy: Along that same vein, I liked her timing. You know, like you said, she left it open so that the students could get their questions out and the attempts of the youngster to try to explain what she had on the board. But then didn't let it go too long when she rephrased the question. Or, she actually identified what seemed to be the confusion. Her timing, I thought, was very good.

Facilitator #1: I guess that's two different things, to be able to reframe and also to identify a confusion.

Donna: And actually the first thing that she did was great. They were starting to criticize the work. And I was in a classroom the other day where the same thing happened -- it was something that there could be many answers and the teacher stopped and just like this teacher and said, "Don't criticize. What can you say that asks it as a question?" And so immediately the kids reframed it. Which is what they should have been doing as a question. And, if she hadn't done that early on, the whole thing would have deteriorated and the two kids up there would have felt terrible what they had done. So, right from the start, doing that -- questioning -- made it rather than a [gets a little stuck]//

Facilitator #1: Yea, she asked the student to reframe it as a question.

Facilitator #2: My sense was that this must be .. that the kids were good enough at asking each other the questions about the work that that must be general culture of the room.

Donna: But originally they didn't do it [says something else that is inaudible]//

Facilitator #2: She set it up.

Donna: Yes, they could do it .. they were able to do it.

[Someone else: the rephrasing what another kid said]

Facilitator #1: They knew what her question , they knew what she was asking them to do.

Throughout the course, many of the administrators pointed to classroom management and timing behaviors as indicators of good teaching. This is typical of the traditional way to appreciate classrooms. In this scenario, Judy noticed that the teacher had "good timing," in that she had a good sense of how long to let the students try to explain their answer before she stepped in to help them. One can gather from the comment that Judy had a sense of what would feel like "too long" to her to let the confusion go on.

Instead of pursuing the timing issue, the facilitator *repeated* a portion of Judy's comment, "I guess that's two different things, to be able to reframe and also to identify a confusion," highlighting the distinction between reframing (in reference to the administrator's comment about rephrasing the question) and identifying a confusion. In this case, the facilitator's comment served to provide an analytic refinement of administrator's distinction and to put the focus not on the timing issue but on what the teacher had to know to be able to make her timing work: she had to be able to reframe and she had to be able to identify a confusion.

By pulling attention to what was behind the teacher's timing, the facilitator allowed Donna, the math specialist to comment on an earlier move the teacher had made — to ask a student to restate a comment as a question. As Donna noted, "If she hadn't done that early on, the whole thing would have deteriorated and the two kids up there would have felt terrible [with] what they had done." This comment allowed the group to pay attention to the classroom culture. The primary facilitator *paraphrased* the statement, saying "she asked the student to reframe it as a question." One of the other facilitators added her own interpretation: "My sense was ...that [if] the kids were good enough at asking each other the questions about the work, that that must be general culture of the room." When Donna pointed out that the students did not phrase comments as questions on their own, the primary facilitator noted that they "knew what she was asking them to do."

In this scenario, the two facilitators worked to reshape the significance of the timing that Judy highlighted by drawing attention to classroom culture that the teacher constructed through certain classroom strategies that became visible as good timing. Specifically, the facilitators highlighted and valued the teacher's ability to identify confusions, reframe comments and confusions, advise students to change criticisms to questions, and most importantly cultivate these abilities in the students themselves. In this episode we can clearly see the facilitators working to develop a new appreciation for teaching — coming to see it not only as having good timing from a classroom management point of view, but also as including the effort to create a classroom culture that supports mathematical discourse among students.

Conclusions

This paper has illustrated how the facilitators in a professional development program for school and district administrators used a range of discursive moves to help the administrators cultivate a different appreciation of mathematics classrooms and learn to pay attention to different features of a mathematics lesson. While the facilitators' moves can be described in rather straightforward terms – suggesting a different interpretation of a video segment, encouraging a participant to say more about a point, or highlighting a different aspect of the lesson – their salience lies in what they were used to accomplish.

We have shown how these moves were employed strategically to shift administrators' appreciation of mathematics classrooms to ways that placed more value on the mathematical ideas in the lesson and the teacher's mathematical intent. Each of the vignettes illustrated the subtle and often seamless ways in which a facilitator could reshape what was perceived as salient in a particular video clip. The first vignette showed how the facilitators helped to bring to light the nature of mathematical reasoning and the cultivation of number sense. The second vignette showed a facilitator stressing the limitations of thinking about mathematics as a discipline that could be learned through clear, direct, instruction. The third vignette illustrated a facilitator

emphasizing the importance of being able to look beyond the standard classroom markers of student engagement (e.g. who is participating and how often) to look for how students' ideas become part of the classroom discourse and how comfortable students are with taking risks. The fourth vignette showed two facilitators turning an observation principally about classroom management into an illustration of teaching as the facilitation of student inquiry. In each case, we saw important new ideas about mathematics education being developed through reference to concrete images on the videotape.

These vignettes also point to subtle but important implications for facilitating in professional development programs such as the one we describe here. Specifically, while facilitation moves such as the discursive ones we note here can be delineated, it is not enough for someone to have memorized a list of them. As the vignettes highlighted, the moves were used at opportune moments to help administrators rethink what counts as mathematical knowledge, to think about how mathematics is learned, to reconsider what student engagement actually looks like in such classrooms, and to redefine the nature of teaching within such classrooms. As we have illustrated, administrators would raise certain points, often stated within more traditional frameworks of teaching. The facilitators would then build on the ideas and observations that administrators made but would reinterpret or restate them within a reform paradigm.

These observations lead us to conclude that a central task for those who work with school administrators in such professional development contexts is to listen to the ideas about mathematics, learning, and teaching that they bring to the work at hand. Helping administrators shift their appreciative attention requires facilitators to hear in administrators' talk what their views of mathematics, learning, and teaching are. Facilitators also have to be sufficiently sensitized to the variety of ways in which these ideas may be expressed that they can recognize them in the conversational flow. When they can recognize these ideas they are then in a position to help reshape administrators' appreciation.

References

Ball, D. L. 1993. With an eye on the mathematical horizon: Dilemmas of teaching elementary school mathematics. The Elementary School Journal, 93 (4), 373-97.

Ball, D. L. (1997). Developing mathematics reform: What *don't* we know about teacher learning — but would make good working hypotheses. In S. Friel & G. Bright (Eds.) Reflecting on our work. (pp. 77-112). New York: University Press of America.

Brophy, J. & Good, T. L. (1986). Teacher behavior and student achievement. In M. C.. Wittrock (Ed). Handbook of research in teaching, 3rd edition. (pp. 328 - 375). New York: Macmillan Publishing.

Carpenter, T. P., Fennema, E., Peterson, P., & Carey, D. (1988). Teachers' pedagogical content knowledge of students' problem-solving in elementary arithmetic. Journal for Research in Mathematics Education (19), 385-401.

Darling-Hammond, L. & Sclan, E. (1992). Policy and supervision. In C. Glickman (ed.) Supervision in transition: 1992 yearbook of the Association for Supervision and Curriculum Development. (pp. 7-29). Reston, VA: Association for Supervision and Curriculum Development.

Fenstermacher, G. & Richardson, V. (1993). The elicitation and reconstruction of practical arguments in teaching. Journal of Curriculum Studies, 25(2): 100-115.

Glaser, B. & Strauss, A. (1967). The discovery of grounded theory. Hawthorne, NY: Aldine.

Glickman, C. D., Gordon, S. P., & Ross-Gordon, J. M. (1998). Supervision of Instruction: A Developmental Approach. 4th ed. Boston, MA: Allyn and Bacon.

Hammer, D. (1995). Epistemological considerations in teaching introductory physics. Newton, MA. Center for the Development of Teaching, Education Development Center, Inc.

Hunter, M. (1984). Knowing, teaching and supervising. In P. L. Hosford (Ed). Using what we know about teaching. Alexandria, VA: ASCD.

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. American Educational Research Journal 27 (1), 29-63.

Miles, M. B. & Huberman, A. M. (1994). Qualitative data analysis: An expanded sourcebook. (2nd ed.) Thousand Oaks, CA: Sage.

NCTM (1989). Curriculum and evaluation standards for school mathematics. Reston, VA: National Council of Teachers of Mathematics.

NCTM (1991). Professional standards for teaching mathematics. Reston, VA: National Council of Teachers of Mathematics.

NCTM (1998). Principles and standards for school mathematics: Discussion draft. Standards 2000. Reston, VA: National Council of Teachers of Mathematics.

Nelson, B. S., (1998). Lenses on learning: How administrators' ideas about mathematics, learning and teaching influence their approaches to action in an era of reform. Newton, MA: Center for the Development of Teaching, Education Development Center, Inc. Also in the Journal of Mathematics Teacher Education. 1 (2) 191-215.

Nelson, B. S, (1999). Building new knowledge by thinking: How administrators can learn what they need to know about mathematics education reform. Newton, MA: Center for the Development of Teaching, Education Development Center. (An earlier version of this paper was presented as Nelson, B.S., Davidson, E., & Sassi, A,, (1998) Building new knowledge by thinking: How administrators can learn what they need to know about mathematics education reform. Paper presented at the Annual Meeting of the Educational Research Association, San Diego: CA.)

Nelson, B. S., Sassi, A., & Driscoll, M. (1999). Developing a new eye for mathematics classrooms: Classroom observation and teacher supervision. In Journal of mathematics education leadership. 2(4) pp. 4 - 12.

Nelson, B. S. & Sassi, A. M. (1998). Cultivating administrators' professional judgment in support of mathematics education reform: The case of teacher supervision. Paper presented at the annual meeting of the American Educational Research Association, San Diego, CA: April, 1998.

Nolan, J. & Francis, P. (1992) Changing Perspectives in Curriculum and Instruction. In C. Glickman (ed.) Supervision in transition: 1992 yearbook of the Association for Supervision and Curriculum Development. (pp. 7-29). Reston, VA: Association for Supervision and Curriculum Development.

Pendlebury, S. (1995). Reason and story in wise practice. In H. McEwan & K. Egan (Eds). Narrative in teaching, learning, and research. New York: Teachers College Press.

Reitzug, U. C. (1997). Images of principal instructional leadership: From super-vision to collaborative inquiry. Journal of Curriculum and Supervision, 12(4). 356-366.

Schifter, D. & Fosnot, C. T. (1993). Reinventing mathematics education: Stories of teachers meeting the challenge of reform. New York: Teachers College Press.

Schifter, D., & Simon, M. 1992. Assessing teachers' development of a constructivist view of mathematics learning. Teaching and Teacher Education, 8(2), 187-97.

Shulman, L.S. (1986). Those who understand: Knowledge growth in teaching. Educational Researcher, 57(2), 4-14.

Vickers, G. (1965). The art of judgment: A study of policymaking. New York: Basic Books.